Recent Developments in Quantifying Uncertainties and Limitations of 1D Site Response Constitutive Models

JIM KAKLAMANOS

Associate Professor of Civil Engineering Merrimack College North Andover, Massachusetts, USA



MERRIMACK COLLEGE

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1D site response analyses



Output:

Surface ground motion

Analysis method:

- Linear (L)
- Equivalent-linear (EQL, EQL-FD)
- Nonlinear (NL)

Input:

1D Soil profile

- Shear-wave velocity, V_S
- Mass density, ρ
- Small-strain damping ratio, $\xi = D_{min}$
- Additional material parameters
- Input (bedrock) ground motion (downhole or outcrop)

Components of uncertainty in site response modeling

Modified from Rathje et al. (2010) and Passeri (2019)



1. Shear-wave velocity

0

Depth (m)

and density profiles

Vs (m/s)

250 500 750 1000



Gmax

γ1

τ1.



6. Small-strain damping ratio



5. Shear strength and other nonlinear properties

asymptotic to Tr

γ

LOOOOO TGIG

Site response constitutive models

(kPa)

Shear stress,

Stress-strain relationships:

1. Linear:

 $\tau(\gamma) = G_{max}\gamma$, where $G_{max} = \rho V_S^2$.

2. Equivalent-Linear:

 $\tau(\gamma) = G \gamma$, where $G \leq G_{max}$ is determined from an appropriate modulus-reduction relationship.



Stress-strain curves at depth of 2 m for KiK-net site IWTH08

3. <u>Nonlinear</u> (example):

$$\tau(\gamma) = f(\gamma) = \frac{G_{max}\gamma}{1 + \left(\frac{\gamma}{\gamma_r}\right)^{\alpha}},$$

the backbone curve of a hyperbolic-type nonlinear model, where γ_r and α are model parameters. Other types of nonlinear models with different functional forms exist.

Equivalent-linear (EQL) approach

Overview:

- Computed in the frequency domain using iterative linear analyses
- Nonlinearity is modeled by iteratively adjusting the shear modulus G and damping ratio ξ to be consistent with the induced strains ("strain-compatible properties")
- The selected values of G and damping ratio ξ are constant throughout the duration of the loading

Required input parameters:

Soil profile:	Layer depths/thicknesses	-
Small-strain material properties:	Shear-wave velocity, V _s	for each layer in
	Mass density, <i>p</i>	
Dynamic soil behavior characteristics:	Modulus reduction curve, $G/G_{max} = f(\gamma)$	soli
	Damping curve, $\xi = f(\gamma)$	promo
Equivalent-linear computation parameters:	Effective strain ratio, R _y	
	Number of iterations, N	

Equivalent-linear (EQL) approach

Advantages:

- Widely used and understood
- Computationally efficient
- Requires a limited number of input parameters

Disadvantages:

- Involves a significant approximation to fully nonlinear behavior, especially at large strains
- Cannot account for changes in dynamic properties (e.g., G, ξ) throughout the duration of the loading
 - Can result in overdamping (underpredictions) of high frequencies
 - Can result in overpredictions at the fundamental modes of vibration

Equivalent-linear approach with frequency-dependent properties (EQL-FD)

Motivation:

• The traditional equivalent-linear algorithm artificially overdamps high frequencies because it uses the same damping ratio throughout the entire time series (when in fact the damping ratio is compatible with strain levels that occur during a short time interval).



Equivalent-linear approach with frequency-dependent properties (EQL-FD)

Mechanics:

- EQL-FD approaches (e.g., Sugito, 1995; Assimaki and Kausel, 2002; Yoshida et al., 2002) use the complete shear strain frequency spectrum to select strain-compatible properties at each frequency using an iterative procedure.
- Higher-frequency components are more greatly amplified (less damped) in EQL-FD analyses than EQL analyses.

Low-frequency motion is associated with larger strains: $G/G_{max} \downarrow$, Damping \uparrow (greater nonlinearity)

High-frequency motion is associated with smaller strains: $G/G_{max} \uparrow$, Damping \downarrow (less nonlinearity)



Equivalent-linear approach with frequency-dependent properties (EQL-FD)

Advantages:

 More greatly able to overcome the inherent limitations of the traditional EQL approach (constant dynamic properties throughout the entire loading), especially at high frequencies.

Disadvantages:

- Few studies have fully evaluated the performance of EQL-FD analyses.
- Has been shown to overpredict ground motions in the aggregate (e.g., Zalachoris and Rathje, 2015).
- Limited software applications for broad usage, although some options are:
 - SeismoSoil (Asimaki and Shi, 2017)
 - Strata (Kottke and Rathje, personal communication)
 - DYNEQ (Yoshida and Suetomi, 1996)

Nonlinear (NL) approach

Overview:

- Computed in the time domain incrementally by numerically solving the equation of motion at each step
- Dynamic soil parameters can vary throughout the duration of loading, and therefore advanced constitutive models may be implemented

Required input parameters (vary for different constitutive models and software programs):

Soil profile:	Layer depths/thicknesses	_
Small-strain material properties:	Shear-wave velocity, V _s	Often estimated by fitting to modulus reduction and damping curves
	Mass density, <i>p</i>	
Dynamic soil behavior characteristics:	Nonlinear soil model parameters (e.g., D _{min} , coefficients)	
	Hysteretic reloading/unloading formulation parameters	
	Shear strength parameters (if needed)	
	Pore water pressure parameters (if needed)	
Nonlinear computation parameters:	Viscous damping definition (e.g. Rayleigh damping)	
	Time domain computation parameters (e.g. step control, integration scheme, interpolation method)	

Nonlinear (NL) approach

Advantages:

- More accurate representation of dynamic soil behavior (including shear strength) at large strains
- Because soil properties can vary with time, advanced constitutive models may be implemented
- Computation of pore pressures (effective-stress analyses) and permanent deformations are possible

Disadvantages:

- Greater computational effort (especially for batch analyses), although this has become less of an issue over time
- Additional input parameters are usually required
- Numerical errors may occur at high frequencies due to frequencydependent viscous damping and/or integration schemes

Methods of comparison:

- Comparisons to observations at vertical seismometer arrays
- Comparisons between EQL and NL predictions



Literature review:

- Comparisons to observations at vertical seismometer arrays:
 - Andrade and Borja (2006): Lotung, Taiwan, array
 - Lee et al. (2006): Multiple depths for the Lotung, Taiwan, array
 - Assimaki et al. (2008): 3 sites in the Los Angeles basin
 - Stewart et al. (2008): 4 sites (the La Cienega and Turkey Flat arrays in California, Lotung array in Taiwan, and one site in Japan's Kiban-Kyoshin network [KiK-net])
 - Kwok et al. (2008): Turkey Flat array in Parkfield, California
 - Kim and Hashash (2013): 9 KiK-net sites that recorded the 2011 M 9.0 Tohoku-Oki earthquake
 - Yee et al. (2013): nuclear power plant in Japan
 - Kaklamanos et al. (2015): 6 KiK-net sites

Literature review:

- Comparisons to observations at vertical seismometer arrays (cont.):
 - Zalachoris and Rathje (2015): 11 sites (9 KiK-net sites, the La Cienega array in California, and the Lotung array in Taiwan)
 - Du and Pan (2016): 2 sites in Singapore
 - Griffiths et al. (2016): Treasure Island array in San Francisco Bay
 - Régnier et al. (2016, 2018): 2 sites in Japan as part of the PRENOLIN project
 - Shi and Asimaki (2017): 9 KiK-net sites
 - Aristizábal et al. (2018): Euroseistest site in Greece
 - Kaklamanos and Bradley (2018): 114 KiK-net sites
 - Li et al. (2018): 7 arrays in California and Japan

Note: Bolded studies will be discussed in greater detail.

Literature review:

- Comparisons between EQL and NL predictions:
 - Hartzell et al. (2004)
 - Park and Hashash (2005)
 - Snow (2008)
 - Rathje and Kottke (2011)
 - Assimaki and Li (2012)
 - Papaspiliou et al. (2012)
 - Kim et al. (2013, 2016)
 - Faccioli et al. (2015)
 - Carlton and Tokimatsu (2016)
 - Pruiksma (2016)
 - Eskandarinejad et al. (2017)

Results for an example groundmotion record (Kaklamanos and Bradley, 2018): acceleration time series, Husid plots, response spectra [and residuals], and amplification spectra



Applicable ranges of EQL and NL site response models

Studies for discussion:

- Kaklamanos and Bradley (2018, BSSA): "Challenges in predicting seismic site response with 1D analyses: Conclusions from 114 KiK-net vertical seismometer arrays," advancing upon Kaklamanos et al. (2013, 2015).
- Zalachoris and Rathje (2015, JGGE): "Evaluation of onedimensional site response techniques using borehole arrays"
- Shi and Asimaki (2017, BSSA): "From stiffness to strength: Formulation and validation of a hybrid hyperbolic nonlinear soil model for site-response analyses"
- Carlton and Tokimatsu (2016, EQS): "Comparison of equivalent linear and nonlinear site response analysis results and model to estimate maximum shear strain"
- Kim et al. (2016, EQS): "Relative differences between nonlinear and equivalent-linear 1-D site response analyses," advancing upon Kim et al. (2013)

Comparisons between models and to observations at vertical seismometer arrays

Comparisons between EQL-NL model predictions without observations

Kaklamanos and Bradley (2018, BSSA)

Study location and models considered: Linear (L), equivalent-linear (EQL), and nonlinear (NL) analyses of 5626 ground-motion records at 114 KiK-net stations, using SHAKE (Schnabel et al., 1972) and DEEPSOIL (Hashash et al., 2016)



Kaklamanos and Bradley (2018, BSSA)

Quantification of uncertainty: Using mixed-effects regression on the model residuals (for surface ground-motion intensity measures), the L, EQL, and NL model biases and standard deviations (intra-site and inter-site) can be quantified

Mixed-effects regression:

$$y_{i,j} = a + \eta_{S_i} + \epsilon_{i,j}$$

where:

 $y_{i,j} \sim N(a, \sigma_Y^2) = \text{model residual}$ = $\ln(\text{IM}_{\text{obs}}) - \ln(\text{IM}_{\text{pred}})$ a = fixed effect (model bias) $\eta_{S_i} \sim N(0, \tau_S^2) = \text{inter-site residual}$ $\epsilon_{i,i} \sim N(0, \sigma_0^2) = \text{intra-site residual}$

Model bias (mean residual)

(b) ≿ (a) 0.6 0.7 Underprediction Ø Total standard deviation, 0.6 Fixed effect (bias), 0.4 0.5 0.2 0.4 0.3 0.0 0.2 inear Equivalent-Linear 0.1 -0.2 Overprediction Nonlinear 0.0 0.01 0.1 3 0.01 0.1 3 Spectral period, T (sec) Spectral period, T (sec)

Kaklamanos and Bradley (2018)

Total standard deviation

Key conclusion: All models are biased towards underprediction of ground motions at high frequencies (short spectral periods), where nonlinear effects are strongest.

Kaklamanos and Bradley (2018, BSSA)

Analysis of intra-site residual plots:

Allow for the determination of strain thresholds at which models fail to be accurate

Vertical axis:

L, EQL, and NL intra-site residuals ($\epsilon_{i,j}$) for PGA; PSA at T = 0.1, 0.2, and 0.5 s; and Arias Intensity

Horizontal axis: Maximum shear strain, γ_{max}

Kaklamanos and Bradley (2018)



Kaklamanos and Bradley (2018, BSSA)

Usage of Husid Intensity to differentiate between EQL and NL models: For $\gamma_{max} \ge 0.05\%$, the EQL model is shown to have excessive bias early in the ground motion record, but this bias is obscured when the entire record is considered.



Kaklamanos and Bradley (2018, BSSA)

Ranges of applicability of L, EQL, and NL site response models:



- At different spectral periods, deviations of the intra-site residual trendlines from zero were used to establish period-dependent ranges at which each model remains applicable.
- Building on the recommendations of Kaklamanos et al. (2013), for short spectral periods:
 - Linear analyses begin to lose accuracy in the 0.01% < γ_{max} < 0.1% range.
 - EQL and NL analyses begin to lose accuracy in the $0.1\% < \gamma_{max} < 0.4\%$ range, although the NL thresholds are larger (up to $\gamma_{max} = 1\%$) for spectral periods in the 0.03 < T < 0.1 s range.
- When Husid Intensity is considered, however, NL models are shown to have a significant advantage over EQL models are smaller strain ranges, e.g. $\gamma_{max} \ge 0.05\%$, at short periods.

Kaklamanos et al. (BSSA, in review)

Potential bias due to overly coarse V_s profiles:

- **Hypothesis:** The V_S profiles provided on the KiK-net website may be too coarse, and the impedance contrasts between successive layers may be larger than those in reality.
- Action: Within each layer, the constant value of V_S is replaced with a depth-dependent exponential gradient centered on the median V_S for the layer.
- **Result:** The application of the V_S gradient results in greatly reduced high-frequency bias, implying that coarse V_S profiles are partially responsible for the underprediction bias at high frequencies.



Kaklamanos et al. (in review)

Study location and models considered: Equivalent-linear (EQL), equivalent-linear with frequency-dependent properties (EQL-FD), and nonlinear (NL) analyses of 661 ground-motion records at 11 stations (9 KiK-net sites, the La Cienega array in California, and the Lotung array in Taiwan), using Strata (Kottke and Rathje, 2008) and DEEPSOIL, with and without corrections for large-strain shear strength

Key findings:

- All three site response techniques (EQL, EQL-FD, and NL) were unable to accurately predict site amplification at short periods (up to 0.4 s) for shear strains greater than 0.1%.
- In the aggregate, the EQL and NL approaches tended to underpredict site amplifications, and the EQL-FD approach tended to overpredict site amplifications.
- Corrections for large-strain shear strength slightly improved the underpredictions of EQL and NL models, but did not have an effect on the EQL-FD predictions.





Zalachoris and Rathje (2015)

Computed mean prediction residuals: NL approach



Zalachoris and Rathje (2015)

Computed mean prediction residuals: EQL-FD approach



Zalachoris and Rathje (2015)

Shi and Asimaki (2017, BSSA)

Study location and models considered: Linear (L), equivalent-linear (EQL) using modulus-reduction curves from the MKZ model (Matasovic and Vucetic, 1993) and their proposed Hybrid Hyperbolic (HH) model, and nonlinear (NL) analyses using both the MKZ and HH constitutive models, for 2756 ground-motion records at nine KiK-net sites, using SeismoSoil

Key findings:

- Using a goodness-of-fit (GOF) score that incorporates Arias Intensity, energy integral, spectral acceleration, Fourier spectra, and root-mean-square acceleration/velocity/displacement, the GOF of the models was found to deviate for a shear strains exceeding a threshold level of of $\gamma_{max} = 0.04\%$ (approximately PGA = 0.05g).
- For maximum shear strains beyond 0.04% (and especially beyond 0.1%), the L model severely overpredicted large-strain motions, the EQL-MKZ and NL-MKZ models severely underpredicted large-strain motions (both to similar degrees), and the EQL-HH and NL-HH approaches outperformed the other site response models (with the NL-HH model offering a stronger goodness-of-fit).

Shi and Asimaki (2017, BSSA)



Computed goodness-of-fit scores:

Modified from Shi and Asimaki (2017)

Carlton and Tokimatsu (2016, EQS)

Study location and models considered: Comparison of equivalent-linear (EQL) and nonlinear (NL) model predictions at 16 sites (9 hypothetical, 7 actual) paired with 189 ground motions (from the NGA-West2 database [Ancheta et al., 2014] and from Carlton [2014]).

Key findings:

- The strongest predictor of the difference between EQL and NL predictions was the maximum shear strain induced in the EQL analysis ($\gamma_{max,ELA}$), and a model was developed to predict γ_{max} based on site- and ground-motion parameters that may be known prior to conducting a site response analysis.
- For large shear strains ($\gamma_{max,ELA} > 0.1\%$), EQL analyses predicted larger spectral accelerations than NL analyses for short periods (< 0.1 s) and near the natural periods of the sites investigated (0.2 < T < 2 s). For periods between 0.1-0.2 s, the EQL model predicted slightly larger accelerations.
- Differences in predicted ground-motion parameters are non-negligible for γ_{max} values ranging from 0.05% to 1%, as a function of the spectral period.

Carlton and Tokimatsu (2016, EQS)



Carlton and Tokimatsu (2016)

Kim et al. (2016, EQS)

Study location and models considered: Comparison of equivalent-linear (EQL) and nonlinear (NL) model predictions for 42 sites paired with 321 ground motions (from the NGA-W2 database [Ancheta et al., 2014] and from McGuire et al. [2001] to be representative of active crustal and stable continental regions, respectively).

Key findings:

- The strongest predictor of the difference between EQL and NL predictions was the shear strain index (I_{γ}) , the ratio of PGV_{input} to the site V_{S30}.
- Mean ratios of spectral accelerations (Sa_{EL} / Sa_{NL}) and Fourier spectra (Fa_{EL} / Fa_{NL}) are nearly identical for $I_{\gamma} < 0.03\%$ (approximately $\gamma_{max} < 0.06-0.08\%$). Deviations occur near this strain level for high frequencies, near $I_{\gamma} \approx 0.1\%$ (approximately $\gamma_{max} \approx 0.5\%$) for moderate frequencies (1-3 Hz), and at larger strain levels for low frequencies.
- Frequency-dependent I_{γ} thresholds were developed for the strain levels at which the EQL and NL predictions deviate by 10% to 30%; for the 20% deviation level, $I_{\gamma} = 0.09 f^{-0.8}$, where f < 5 Hz for spectral accelerations and f < 10 Hz for Fourier amplitudes.

Kim et al. (2016, EQS)



Shear strain index, I_{y} (%)

Kim et al. (2016)

Relationship between I_{γ} and γ_{max} :

relationship between I_{ν} and γ_{max} , and they developed a correction factor targeted for profiles with strong impedance contrasts.

Kim et al. (2016, EQS)

Comparisons to other studies:



Kim et al. (2016)

Summary

- Recent studies have shown that EQL analyses begin to lose accuracy at high frequencies for maximum shear strains in the range of 0.04% to 0.1%. NL analyses often show nominally greater accuracy at slightly larger shear strains (up to 0.1% to 0.4%).
- An open challenge with all 1D site response approaches, however, is that all models have been shown to exhibit bias in the aggregate when compared to vertical seismometer array data.
- Other factors besides the selection of the constitutive model type, such as the characterization of the shear-wave velocity profile and material properties, as well as breakdowns in the 1D site response model assumptions, often have a more profound influence on site response model performance.

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