Fragility Functions

• What are fragility functions?
  – Provides probability of incurring a given level of damage based on the imposed demand (usually ground or building response)

![Diagram of fragility functions showing the probability of damage states 1, 2, and 3, with curves defining the probability of incurring damage state 2 or greater, $P[DM \geq DS_2 | Demand]$, and the probability of incurring damage state 2, $P[DM = DS_2 | Demand = X]$.)
Building-level fragility functions

- Fragility functions for an entire building class
  - E.g. mid-rise reinforced concrete buildings

- Usually used in regional-level assessments

- Damage generally assumed to be drift-related
  - E.g. Reinforced concrete frame buildings (HAZUS®)

<table>
<thead>
<tr>
<th>Building Category</th>
<th>Slight</th>
<th>Moderate</th>
<th>Extensive</th>
<th>Complete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-rise</td>
<td>0.50</td>
<td>1.00</td>
<td>3.00</td>
<td>8.00</td>
</tr>
<tr>
<td>Mid-rise</td>
<td>0.33</td>
<td>0.67</td>
<td>2.00</td>
<td>5.33</td>
</tr>
<tr>
<td>High-rise</td>
<td>0.25</td>
<td>0.50</td>
<td>1.50</td>
<td>4.00</td>
</tr>
</tbody>
</table>
### Building-level fragility functions

- Damage-state based on indicators on the extent of deformation in most members
  - E.g. reinforced concrete frame buildings (HAZUS®)

<table>
<thead>
<tr>
<th>Damage State</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slight</td>
<td>• Some structural members exhibit hairline cracks</td>
</tr>
</tbody>
</table>
| Moderate     | • Most members exhibit hairline cracks  
  • Some members have *yielded* and exhibit large flexural cracks and some concrete spalling |
| Extensive    | • Some members have *reached ultimate capacity*, and exhibit large flexural cracks, concrete spalling, and buckled reinforcing |
| Complete     | • Building is in imminent danger of collapse due to *loss of frame stability* |
Building-level fragility functions

• Drifts often correlated back to ground motion intensity
• Examples available in HAZUS®, Uma et al. (2008), among others

Damage to mid-rise reinforced concrete buildings based on spectral displacement (HAZUS®)

Damage to 1977-1992 mid-rise reinforced concrete buildings based on spectral acceleration (Uma et al. (2008))
Component-level fragility functions

- Damage within a specific building depends on several factors, such as but not limited to:
  - Detailing (e.g. beam sizes and reinforcing content)
  - Type (e.g. normal vs low-damage partitions)
  - Density and layout (e.g. less partitions used in open-plan office vs enclosed rooms)

- Component-level fragility functions are therefore commonly used in building-specific assessments
Component-level fragility functions

- Damage-state often based on repair methods required. This allows for easier estimate of repair cost/time.

  – Example by Retamales et al. (2013) for gypsum partitions

<table>
<thead>
<tr>
<th>Damage State</th>
<th>Damage Description</th>
<th>Repair Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superficial damage to walls</td>
<td>Cracks along cornerbeads and joint paper tape, screws pulls out from gypsum</td>
<td>Cosmetic repairs, such as applying joint compound, sanding, and painting</td>
</tr>
<tr>
<td>Local damage of gypsum wallboards and/or steel frame</td>
<td>Crushing of wall corners, out-of-plane bending and cracking of gypsum wallboards,</td>
<td>Local repairs, such as repair or replacement of wallboards</td>
</tr>
<tr>
<td>components</td>
<td>bending of boundary studs</td>
<td></td>
</tr>
<tr>
<td>Severe damage to walls</td>
<td>Tears in steel tracks, track flanges bent, hinges forming in studs, partition will</td>
<td>Replacement of entire wall</td>
</tr>
<tr>
<td></td>
<td>collapse</td>
<td></td>
</tr>
</tbody>
</table>
Component-level fragility functions

- Example of severe wall damage (Reamales et al. (2013))
Component-level fragility functions

- Damage are conditioned to engineering demand parameters (EDP). Examples are:
  - Interstorey drifts (e.g. beams, columns, partitions)
  - Peak total acceleration (e.g. ceiling, services, content)

Graphs showing fragility curves for:
- Full-height commercial gypsum partitions (Retamales et al. (2013))
- Suspended acoustical ceilings (NIBS (1997), Aslani (2005))
Data sources to develop fragility functions

- Experimental
  - Less uncertainty but expensive
  - E.g. Aslani (2005) for interior beam-column joints

<table>
<thead>
<tr>
<th>Specimen #</th>
<th>Reference</th>
<th>Label</th>
<th>IDR_{DS3}</th>
<th>IDR_{DS4}</th>
<th>IDR_{DS5}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hatamoto and Bessho (1988)</td>
<td>F-2</td>
<td>1.20</td>
<td>3.40</td>
<td>5.00</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>F-3</td>
<td>**</td>
<td>**</td>
<td>5.00</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>F-4</td>
<td>1.70</td>
<td>**</td>
<td>5.00</td>
</tr>
<tr>
<td>4</td>
<td>Leon (1990)</td>
<td>BCJ2</td>
<td>2.27</td>
<td>**</td>
<td>5.98</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>BCJ3</td>
<td>1.70</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>BCJ4</td>
<td>1.24</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>28</td>
<td>Yin et al. (2001)</td>
<td>C1A</td>
<td>2.10</td>
<td>3.20</td>
<td>**</td>
</tr>
<tr>
<td>29</td>
<td></td>
<td>C1B</td>
<td>2.00</td>
<td>3.60</td>
<td>**</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td>C4A</td>
<td>2.20</td>
<td>3.85</td>
<td>**</td>
</tr>
<tr>
<td>31</td>
<td></td>
<td>C4B</td>
<td>**</td>
<td>3.80</td>
<td>**</td>
</tr>
</tbody>
</table>

** Information not available
Data sources to develop fragility functions

- Numerical/analytical
  - Used where:
    - Test data on component elements are available (e.g. cross tees), but not for the component itself (e.g. ceilings)
    - Models capable of capturing failure mechanisms are used instead of experiments to reduce cost
  - E.g. Paganotti et al. (2011) for ceiling system fragility
Data sources to develop fragility functions

• Field Data
  – Realistic, though can be limited in number and quality of data
  – E.g. Kaneko and Hayashi (2004) developed fragility functions for rigid bodies overturning based on the percentage of tombstones which toppled in the Hyogo-ken Nambu and Kushiro-Oki Earthquakes

<table>
<thead>
<tr>
<th>Graveyard ID</th>
<th>Area</th>
<th>Overturning ratio (%)</th>
<th>Estimated PGV (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Takarazuka</td>
<td>97</td>
<td>67</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>65</td>
<td>92</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>75</td>
<td>Hiroo</td>
<td>4.73</td>
<td>16.2</td>
</tr>
</tbody>
</table>
Data sources to develop fragility functions

• Expert opinion
  – Using judgement of experts with professional experience in design or post-EQ observation
  – Sample survey by Porter et al. (2007)
Derivation of Fragility Functions

• 6 different approaches (Porter et al. 2007):
  – Actual failure EDP: all specimens failed, EDP known
  – Bounding EDP: some specimens failed, peak EDP known
  – Capable EDP: no specimens failed, EDP known
  – Derived fragility: analytical method
  – Expert opinion: expert judgement
  – Updating: update existing functions using new data
Actual failure EDP

• Used where failure was observed in all specimens, and the EDP which caused the failure is known.

• If $M$ specimens were tested, and $x_i$ is the EDP at which damage was observed to occur in specimen $i$, lognormal mean, $x_m$, and dispersion, $\beta$, calculated as follows:

\[
x_m = \exp \left( \frac{1}{M} \sum_{i=1}^{M} \ln x_i \right)
\]

\[
\beta = \sqrt{\frac{1}{M - 1} \sum_{i=1}^{M} \left( \ln \left( \frac{x_i}{x_m} \right) \right)^2}
\]
Actual failure EDP

- Example
  - Aslani (2005) provides interstorey drifts at which 43 pre-1976 reinforced concrete slab-column connections experienced cracking of no more than 0.3 mm width.

Distribution can be checked against goodness-of-fit tests (e.g. Kolmogorov-Smirnov or Lilliefors tests) at a given significance level.
Bounding EDP

- Used where some specimens were damaged, and the maximum EDP that each of the specimens is subjected to is known.

- Steps consists of:
  - Grouping specimens into bins by max EDP range
  - Calculating the inverse probability of failure for each bin
  - Plotting inverse probability versus average max EDP within each range on lognormal probability paper
  - Fitting a line $\hat{y} = s \ln(x) + c$ through the data points
  - Median: $x_m = \exp\left(-\frac{c}{s}\right)$
  - Dispersion: $\beta = \frac{1}{s}$
Bounding EDP

- Example (Porter et al. (2007))

<table>
<thead>
<tr>
<th>Bin</th>
<th>EDP range</th>
<th>Average (x)</th>
<th>Number of data points (M)</th>
<th>Number of failures (m)</th>
<th>Natural log of average (ln(x))</th>
<th>Inverse probability (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15-0.25</td>
<td>0.2</td>
<td>52</td>
<td>0</td>
<td>-1.61</td>
<td>-2.08</td>
</tr>
<tr>
<td>2</td>
<td>0.25-0.35</td>
<td>0.3</td>
<td>48</td>
<td>4</td>
<td>-1.20</td>
<td>-1.27</td>
</tr>
<tr>
<td>3</td>
<td>0.35-0.45</td>
<td>0.4</td>
<td>84</td>
<td>8</td>
<td>-0.92</td>
<td>-1.25</td>
</tr>
<tr>
<td>4</td>
<td>0.45-0.55</td>
<td>0.5</td>
<td>35</td>
<td>15</td>
<td>-0.68</td>
<td>-0.14</td>
</tr>
<tr>
<td>5</td>
<td>0.55-0.65</td>
<td>0.6</td>
<td>41</td>
<td>12</td>
<td>-0.51</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

\[ y = \Phi^{-1} \left( \frac{m + 1}{M + 1} \right) \]

\[ x_m = \exp \left( -\frac{0.53}{1.60} \right) = 0.72 \]

\[ \beta = \frac{1}{1.60} = 0.63 \]
Derived Fragility Function Method

• Capacity of some components can be calculated by modeling the component as a structural system, and determining the EDP that could cause the system to reach the damage level of interest.

• If \( x \) is the calculated capacity:
  \[
  x_m = 0.92x \quad \beta = 0.4
  \]

• An alternative approach is to use an fault-tree analysis or Monte Carlo simulations
  – E.g. repeat analysis numerous times by changing the property of some variables (e.g. yield strain of reinforcing bars)
Other considerations

- Mechanics approach
  - A more “case-specific” fragility function may be to calculate the median value using mechanics
  - E.g. yield drift of a cantilever steel column can be calculated easily.

\[
\Delta H_{\text{Drift}} = \frac{M_y H^2}{3EI}
\]

Calculating yield drift of cantilever steel column:
Other considerations

• Must check if assumed distribution (i.e. lognormal distribution) fits data
  – E.g. Kolmogorov-Smirnov test: error between data and best-fit curve must be within allowable tolerance (based on data size, confidence level, etc)

Good-fit example [Aslani (2005)]

Poor-fit example
Other considerations

• Fragility functions may cross (example a). Indicates that probability of incurring DS2 is negative (not sensible).

• Two methods of adjustment:
  – Set DS2 and DS3 probability to be equal at affected range (example b)
  – Revise fragility functions (example c)

Porter et al. (2007)
Other considerations

• Correlation may be required between fragility groups
  – Example 1: Mechanical services (e.g. HVAC systems) may be damaged in an earthquake and collapse onto ceilings.
  – Example 2: Partitions may be used to brace ceilings (and vice versa), and hence one can damage the other

Dhakal and MacRae (2013)

Failure of wall braces could have easily caused damage to ceiling and services too

Duct failure may have contributed to the failure of ceiling as well

Dhakal and MacRae (2013)
1. What types of low-damage systems are being developed/used in New Zealand?

2. Are there experimental results or ongoing tests which may be used to develop fragility functions applicable for New Zealand conditions, particularly for low-damage systems?

3. Which method of fragility curve development is appropriate for each low-damage system?

4. What field data from recent New Zealand earthquakes can be used to develop fragility functions for New Zealand conditions?