Structural collapse capacity estimation

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QuakeCoRE OpenSees Workshop 2017

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CHRISTCHURCH NEW ZEALAND

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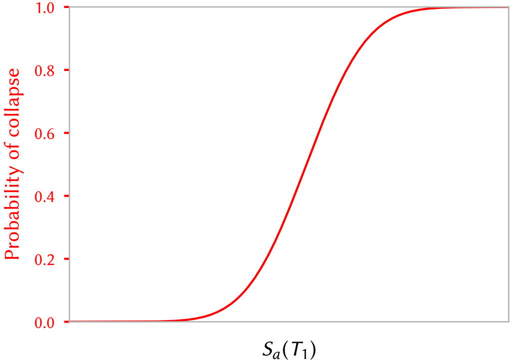
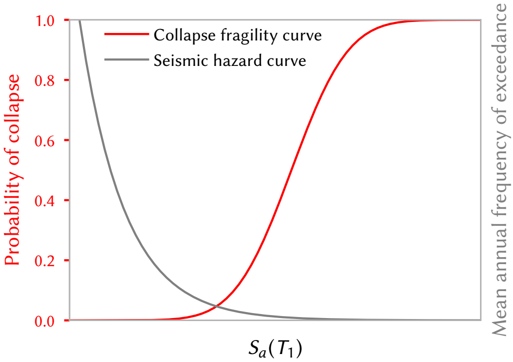
What is structural collapse capacity?



- Capacity of a structure to resist collapse under earthquake ground motion
- Defined by a *collapse fragility curve*, which quantifies the probability of collapse as a function of the ground motion intensity, typically represented by $S_a(T_1)$
- Distinct from traditional measures of structural capacity under equivalent static loading, e.g., base shear capacity and ductility capacity

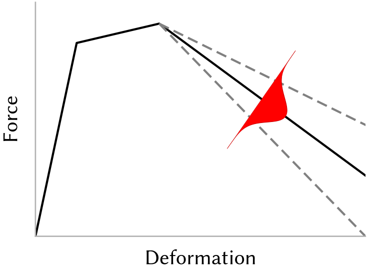
Why do we need to estimate it?



- Gives the *mean annual frequency of collapse* when integrated with the seismic hazard curve
- Primary objective of building design codes is to minimize the probability of structural collapse; hence it is used in design code calibration
- Integral component of seismic loss assessment studies

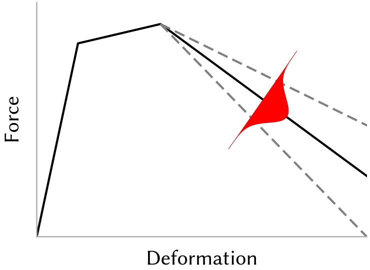
Considerations when estimating collapse capacity

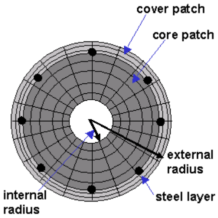
Structural models



- Should use structural models that are capable of faithfully simulating structural response at large inelastic deformations
- Ideally, all deterioration and collapse modes should be explicitly modelled
- If not possible, requires consideration of implicit non-simulated collapse modes
- Uncertainty in model parameters should be accounted for

Plastic hinge vs. fibre models

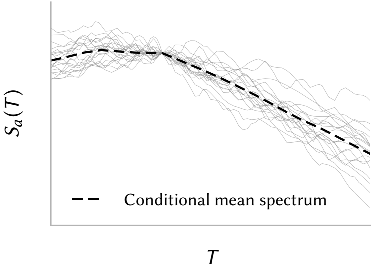




- Concentrated plastic hinge models can implicitly capture deterioration modes like rebar slip and buckling in concrete members, and local flange and web buckling in steel members
- Distributed plasticity fibre element models cannot explicitly capture these phenomena because of plane-sections-remain-plane assumption

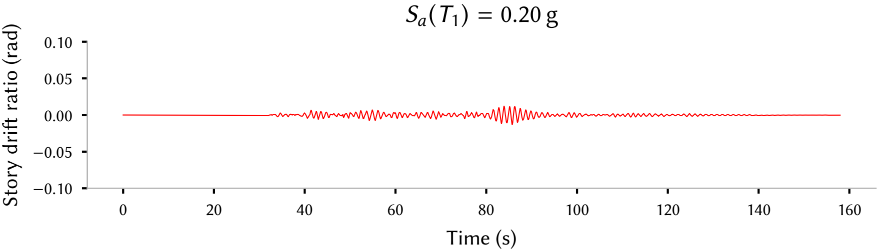
Considerations when estimating collapse capacity

Ground motions



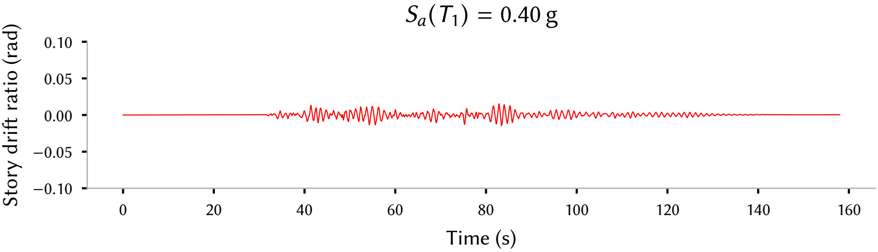
- Requires the numerical simulation of structural response under a large number of ground motions of varying intensities (Scope for parallelisation?)
- Care should be taken to select ground motions with response spectral shapes and durations that are consistent with the seismic hazard at the site
- Uncertainty in the characteristics of the anticipated ground motions should be duly accounted for
- Commonly used analysis methods include *incremental dynamic analysis (IDA)* and *multiple stripe analysis (MSA)*

Incremental dynamic analysis (IDA)



- Uses one set of ground motions
- Each ground motion is progressively scaled to higher intensity levels until it causes structural collapse

Incremental dynamic analysis (IDA)



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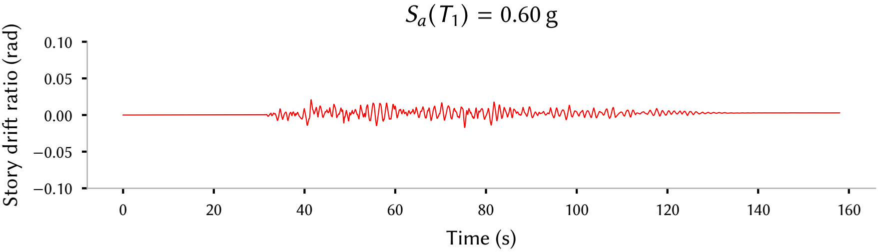

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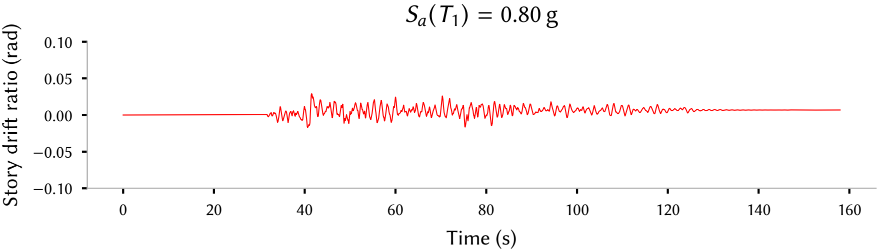

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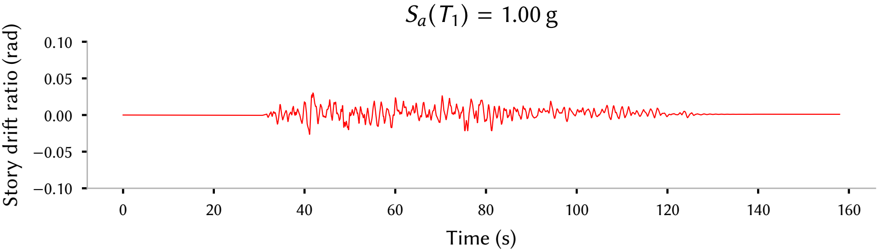

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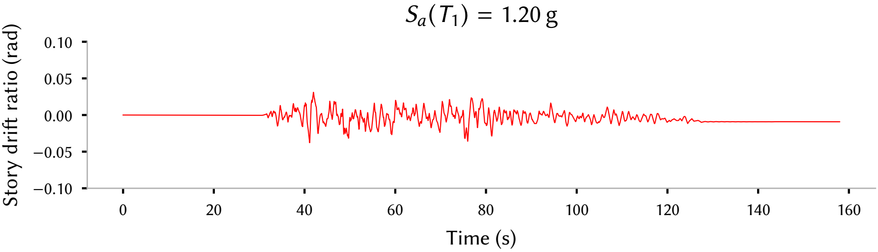

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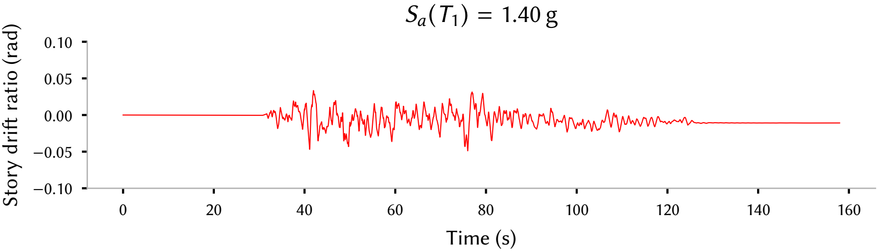

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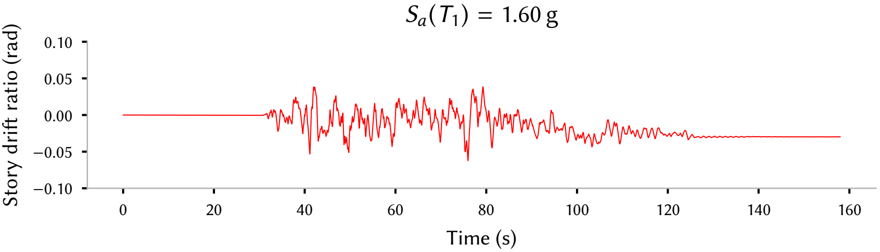

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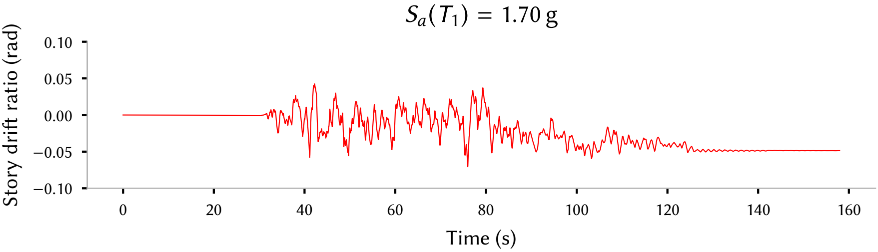

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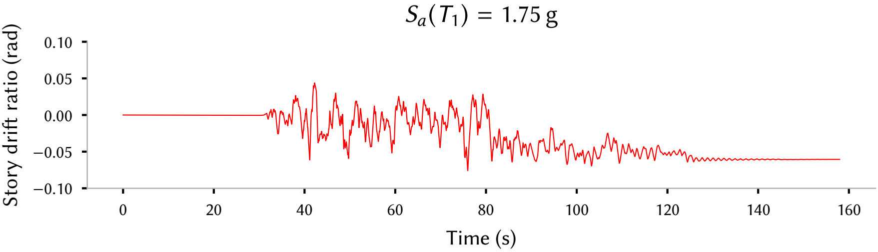

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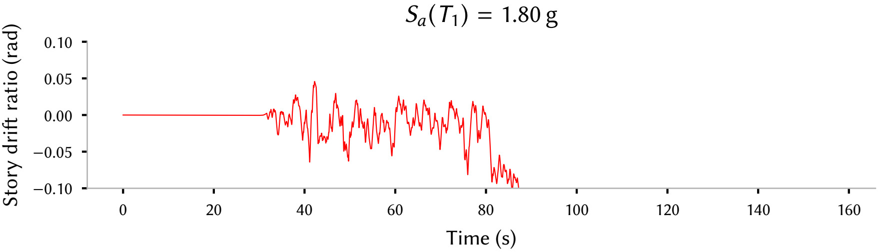

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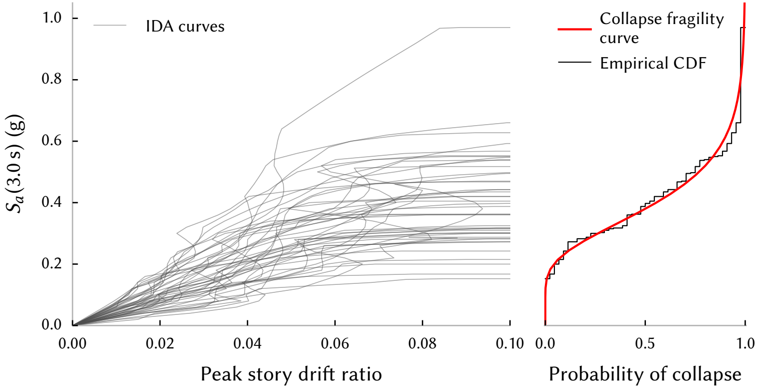
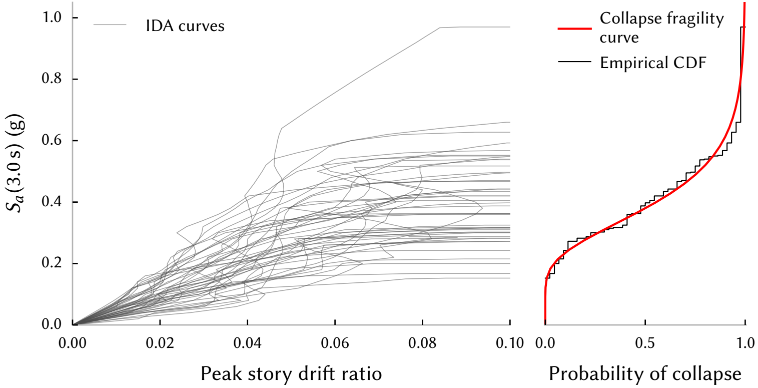


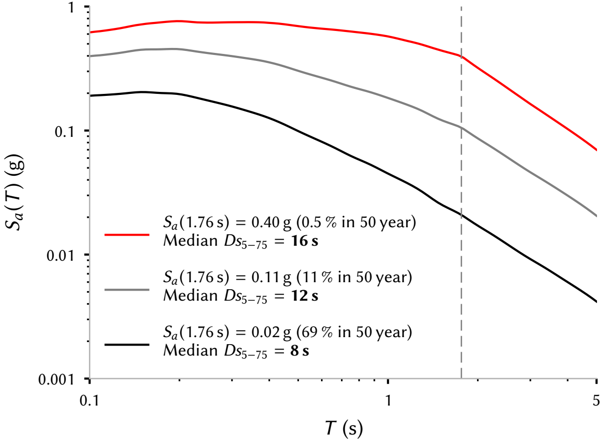
- A plot of the peak story drift ratio observed at each intensity level is called an IDA curve
- 9-story steel moment frame example
  - Video of response just below the collapse intensity
  - Video of response at the collapse intensity

Incremental dynamic analysis (IDA)



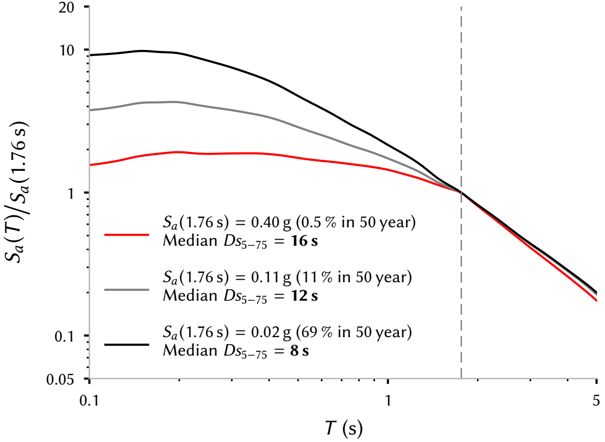
- The variation in the ground motion intensity levels at which structural collapse occurs is due to differences in ground motion characteristics other than $S_a(T_1)$, e.g., response spectral shape and duration
- The collapse fragility curve is fit to the distribution of ground motion collapse intensities

Limitations of IDA



- Cannot capture variation in expected characteristics of ground motions of different intensities, as indicated by the conditional mean spectra and median target durations at Seattle, conditional on different intensity levels
- Hence, the computed collapse fragility curve is not *hazard-consistent*

Limitations of IDA



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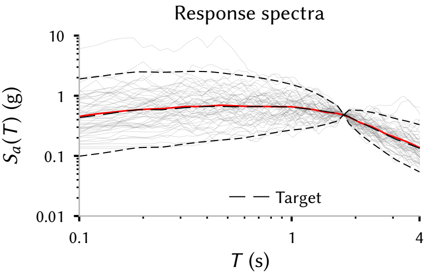
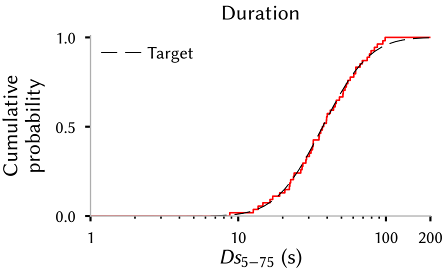
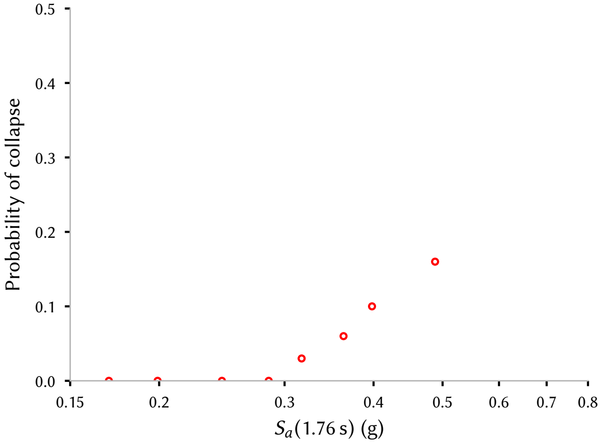
Multiple stripe analysis (MSA)





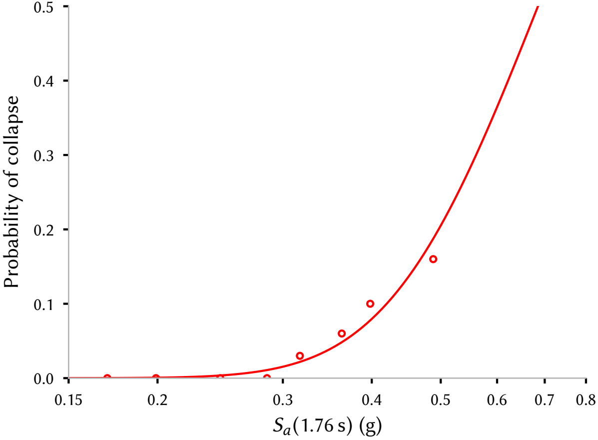
- Different sets of ground motions are selected to match hazard-consistent targets computed at different intensity levels

Multiple stripe analysis (MSA)



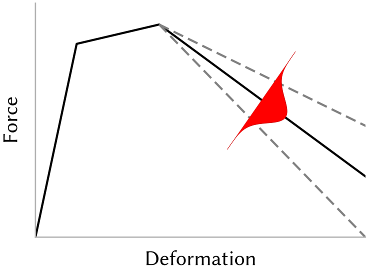
- The structure is analysed under all the selected sets of ground motions
- The probability of collapse at each intensity level is estimated as the fraction of the ground motions at that intensity level that caused structural collapse

Multiple stripe analysis (MSA)



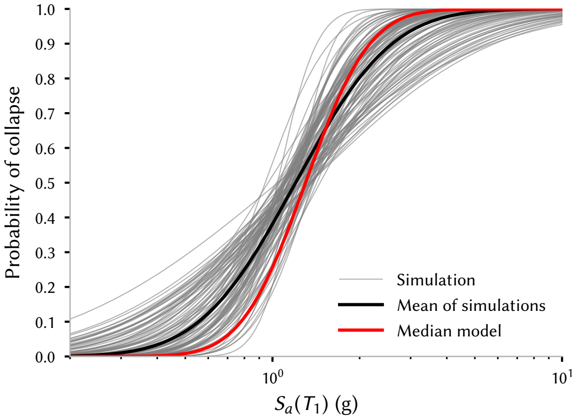
- The collapse fragility curve is then fit through these data points
- This fragility curve is hazard-consistent, unlike that computed using IDA

Accounting for uncertainty in structural models



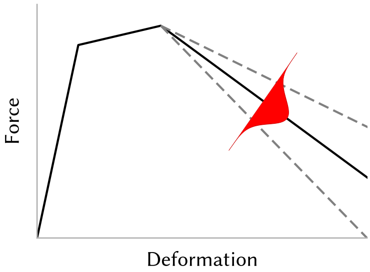
- Requires knowledge of the probability distributions of various model parameters and the correlations between them
- Perform Monte-Carlo simulations to obtain a number of different sets of structural model parameters

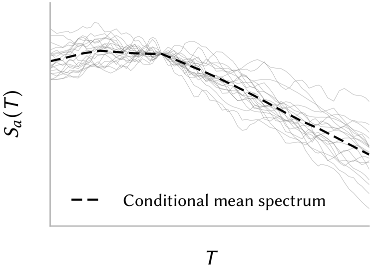
Accounting for uncertainty in structural models



- For each set of structural model parameters, compute a collapse fragility curve (Scope for parallelisation?)
- Compute the mean of all these collapse fragility curves
- This mean fragility curve is likely to be different from the one computed using median model parameters

Solving the **right problem**





- Structural characteristics
  - Use accurate structural model
  - Account for uncertainty in model parameters

- Ground motion characteristics
  - Use hazard-consistent ground motions with appropriate response spectral shapes and durations
  - Account for uncertainty in characteristics of anticipated ground motions

Solving the **problem right**

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{f}(t) = -\boldsymbol{M}\boldsymbol{\iota}\ddot{\boldsymbol{u}}_{\boldsymbol{g}}(t)$$

- Numerical time integration scheme used
  - Implicit schemes (e.g. Newmark average acceleration, HHT-α) often fail to converge, especially when using complex structural models and long duration ground motions
  - Explicit schemes (e.g. central difference) are more robust, and preferred in analyses involving large nonlinear deformations, like blast and crash simulations; structural collapse simulations fall in the same category
- Analysis software (e.g. OpenSees, Perform 3D) and linear algebra solver (e.g. LAPACK, MUMPS, PETSc) used
  - Treatment of ill-conditioned matrices at large nonlinear deformations
- Architecture of machine used to run the analysis
  - Precision of computations

Numerical time integration schemes

- Structural analysis simulations commonly employ implicit time integration schemes
  - They often fail to converge, especially when using long duration ground motions
  - Lots of execution time is spent in attempts to force convergence, which are not always successful
- The explicit central difference time integration scheme is a robust and efficient alternative

Newmark average acceleration vs. Central difference

Newmark average acceleration

Implicit scheme

$$\left(\frac{4}{\Delta t^2}\boldsymbol{M} + \frac{2}{\Delta t}\boldsymbol{C}\right)\boldsymbol{u}_{i+1} = \boldsymbol{p}[\boldsymbol{M}, \boldsymbol{C}, \Delta t, \boldsymbol{u}_i, \dot{\boldsymbol{u}}_i, \ddot{\boldsymbol{u}}_i, (\ddot{\boldsymbol{u}}_g)_i] - \boldsymbol{f}_{i+1}$$

- Solves for equilibrium at end of time step
- Requires solution by iteration; convergence is not guaranteed
- If convergence fails
  - try other solution algorithms, e.g. Modified Newton-Raphson, Newton-Raphson with initial stiffness
  - try other implicit schemes with algorithmic damping
  - try reducing Δt
- These attempts are time-consuming
- If they all fail, structural collapse is declared even if collapse deformation threshold is not exceeded

Central difference

Explicit scheme

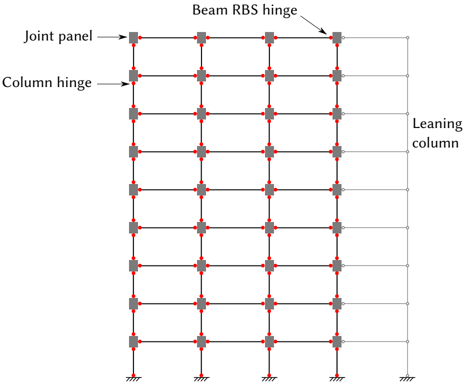
$$\left(\frac{1}{\Delta t^2}\boldsymbol{M} + \frac{1}{2\Delta t}\boldsymbol{C}\right)\boldsymbol{u}_{i+1} = \boldsymbol{p}[\boldsymbol{M}, \boldsymbol{C}, \Delta t, \boldsymbol{u}_i, \boldsymbol{u}_{i-1}, (\ddot{\boldsymbol{u}}_g)_i] - \boldsymbol{f}_i$$

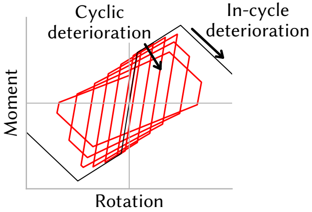
- Solves for equilibrium at beginning of time step
- No iteration required
- If $\boldsymbol{C}$ is constant (and diagonal), matrix needs to be factorized only once
- Very amenable to parallelization by domain decomposition

Newmark average acceleration vs. Central difference

Newmark average acceleration	**Central difference**
Unconditionally stable	Conditionally stable
• Δt limited by accuracy, not stability • Can use relatively large Δt ($\sim 10^{-3}$ s to 10^{-2} s), which is usually reduced upon encountering non-convergence • Not easy to predict duration of analysis	• $\Delta t \leq \frac{T_{min}}{\pi}$ for stability • Δt used is usually relatively small ($\sim 10^{-4}$ s) • T_{min} is usually unchanged in inelastic range • Mass/moment of inertia should be assigned to all degrees of freedom • Impractical to use rigid elements or penalty constraints • Easy to predict duration of analysis (useful for parallel task scheduling)

Structural model

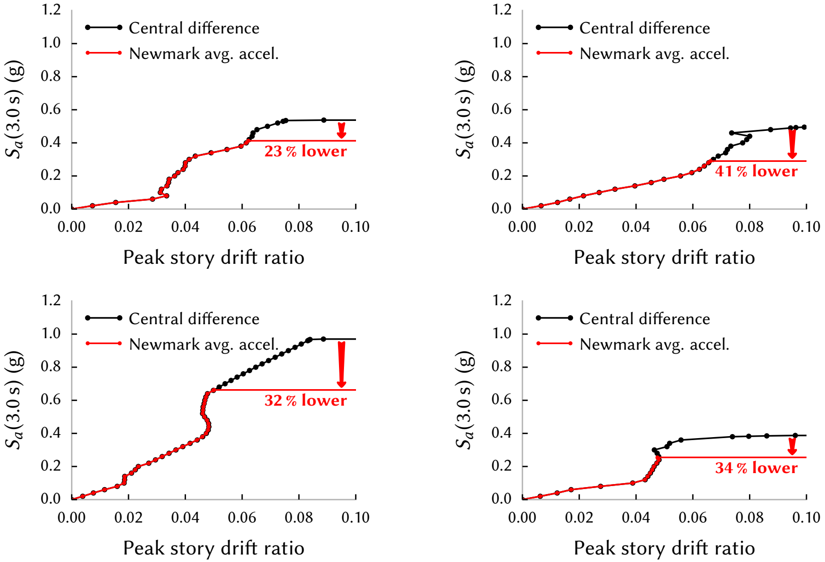




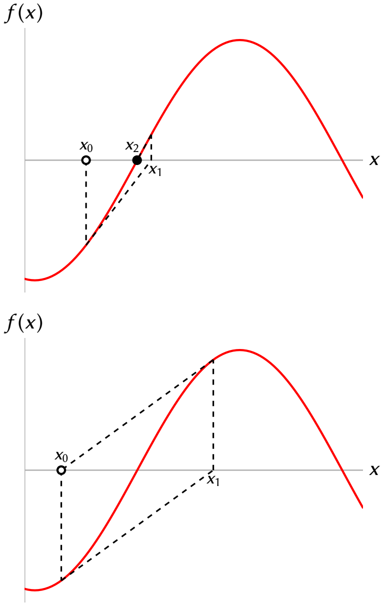
- 9-story steel moment frame building from SAC steel project
- 2d concentrated plastic hinge model created in OpenSees
- Plastic hinges follow Ibarra-Medina-Krawinkler bilinear hysteretic model
- Fundamental elastic modal period is 3.0 s
- Collapse capacity estimated separately using Newmark average acceleration and central difference schemes

IDA curves bifurcate due to non-convergence

Difference in estimated collapse capacity > 10 % for **12** out of 44 ground motions

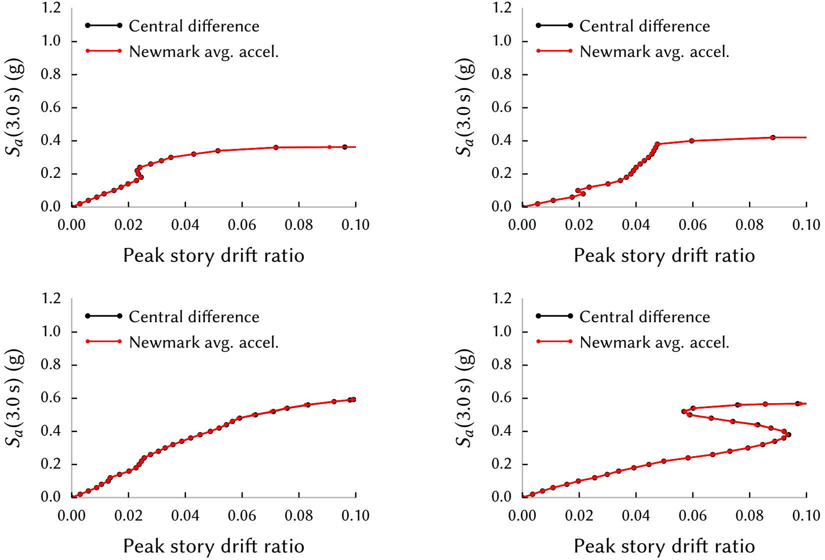


Possible outcomes of *full* Newton-Raphson algorithm

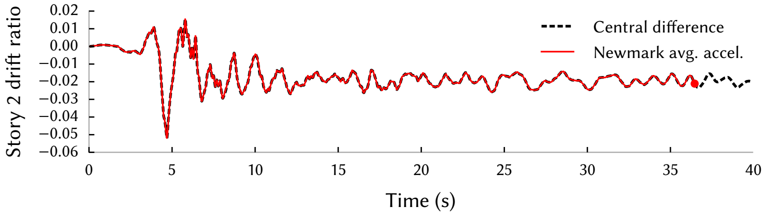


IDA curves are similar when analyses converge

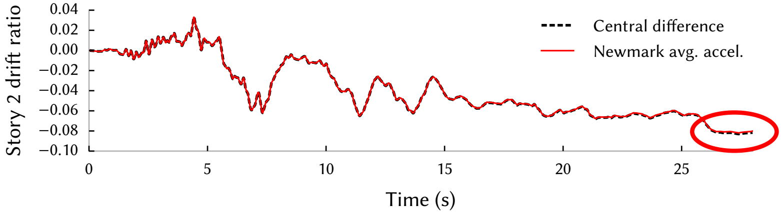
Difference in estimated collapse capacity < 1 % for **29** out of 44 ground motions



Comparison of representative time histories

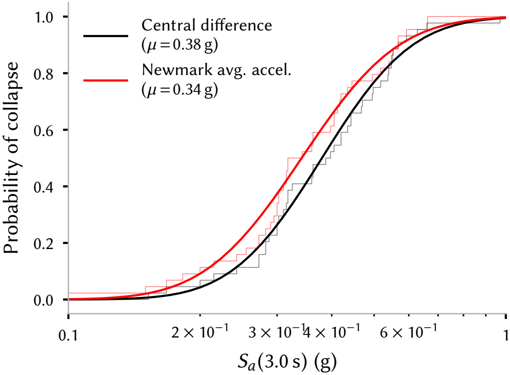


- Time histories are practically identical until the point of non-convergence, if any
- Could use implicit scheme until point of non-convergence and explicit scheme thereafter, but currently facing implementation issues in OpenSees



- Small differences are sometimes observed at large deformations (peak story drift ratio > 0.06)

Effect on estimated collapse fragility curves



- Median collapse capacity is under-estimated by 10 % when using the Newmark average acceleration time integration scheme
- Similar effect expected on collapse fragility curves estimated using multiple stripe analysis as well

Comparison of analysis runtimes

One analysis using one ground motion

Time integration scheme	**Rayleigh damping matrix**	**Type of solver**	Δt (s)	**Analysis runtime** (min)
Newmark avg. accel. low scale factor w/o convergence attempts	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	50×10^{-4}	1.0
Newmark avg. accel. high scale factor w/ convergence attempts	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	$\leq 50 \times 10^{-4}$	20.9
Central difference	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	1.5×10^{-4}	15.9
Central difference	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{initial}$	Sparse (*factor once*)	1.5×10^{-4}	3.3
Central difference	$\alpha \boldsymbol{M}$	Diagonal (*factor once*)	1.5×10^{-4}	2.9

- Using $\boldsymbol{K}_{initial}$ instead of $\boldsymbol{K}_{current}$ in the Rayleigh damping matrix has been shown to produce spurious damping forces
- Other option is to use a modal damping matrix, which is also constant

Comparison of analysis runtimes

Entire IDA (using 160 processors and dynamic load balancing)

Time integration scheme	Rayleigh damping matrix	Type of solver	Δt (s)	IDA runtime (min)
Newmark avg. accel.	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	$\leq 50 \times 10^{-4}$	118
Central difference	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	1.5×10^{-4}	154
Central difference	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{initial}$	Sparse (*factor once*)	1.5×10^{-4}	32
Central difference	$\alpha \boldsymbol{M}$	Diagonal (*factor once*)	1.5×10^{-4}	27

- Only 1 out of 632 total analyses conducted using the Newmark average acceleration scheme completed without any convergence errors
- 567 out of the 632 analyses completed using solution algorithms other than *full* Newton-Raphson
- 23 out of the 632 analyses completed after reducing Δt

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Summary

- Accurate collapse capacity estimation requires
  - Structural model that can faithfully simulate response at large inelastic deformations
  - Selection of ground motions at different intensity levels that are representative of the seismic hazard at the site
  - Consideration of the uncertainty in the structural model and the characteristics of the anticipated ground motions

- The explicit central difference time integration scheme is a robust and efficient alternative to commonly used implicit time integration schemes like Newmark average acceleration

- Advantages of the central difference scheme
  - Robust: not affected by convergence errors
  - Efficient: shorter runtimes despite using a smaller Δt
    - ★ Most efficient when using constant (and diagonal) $\boldsymbol{C}$ matrix
    - ★ Very amenable to parallelization

- Disadvantages of the central difference scheme
  - Mass/moment of inertia should be assigned to all degrees of freedom
  - Impractical to use rigid members or penalty constraints

Thank you!