

# Synthetic study of FWT for a 1-D velocity model

## Problem Statement

The first FWT case study will be a synthetic test with relatively low compute cost to verify the employed methodology. In this case study, we use a 1-D velocity model as the true model and attempt to recover it from an initial homogeneous model. The computational domain will be 60km X 40km X 20km with a test configuration including 12 receivers (3 X 4, equally spacing at every 10km) and 6 sources (2 X 3, equally spacing at every 10km and uniformly embedded at 10km depth). The wavefields are generated by numerical solutions of the 3-D elastodynamic/ visco-elastodynamic equations according to a specific velocity model. The synthetic "observed" data will be generated using the 1-D velocity model (treated as the true model) and synthetic data will be generated using the homogeneous model initially, and the current model in subsequent iterations. The misfit between the "observed" and synthetic data will then be calculated and used in the inversion. After a number of inversion iterations, the final inverted model using FWT is expected to match the 1-D velocity model well.

## Methodology

### Forward modeling of the elastic wave propagation for the 1-D model to create the synthetic "observed" data.

- Choosing a 1-D velocity model as the true model.
- Choosing the cell size for the staggered grid using in forward simulation (0.2 km) and a stable time step according to the CFL criteria.
- Specifying station and source locations; recorded components of the wavefields at the station locations.
- Describing the source signal: using a Ricker wavelet with the central frequency of .5Hz, specifying the recorded time and the time delay for the source.
- Forward modeling of the elastic wave propagation for the 1-D model to create the synthetic "observed" data at receivers.

We can save the velocity wavefields including 3 components or the deformation fields including 9 components. See the PDF file included for more details in formulation of the elastic wave equations\* and how to store the wavefields.

[Elasticwave\\_Eqs.pdf](#)

### Inversion: specify the number of iterations, optimization method or optimal step length (if applicable) and follow the steps for doing inversion at one iteration:

- Forward modeling of the elastic wave propagation for the homogeneous model to create the synthetic data and storing the forward velocity wavefields at every grid cell and for every subsampled time step.
- Calculate the adjoint source\* (multi-channel source at all observing stations) by implementing the cross-correlation between the "observed" and synthetic data using an isolated Gaussian filter. [Adjoint\\_source.pdf](#)
- Backward propagation of the adjoint source at all stations for each and every source and store the backward velocity wavefields at every grid cell and for every subsampled time step.
- Calculation of the sensitive kernels\* (using 3 components of the ground motion velocity wavefields or 9 components of the deformation fields). [Kernels\\_Calculation.pdf](#)
- Preconditioning of the kernels to suppress the near source and receiver artifacts.
- Update of the model with the optimal search direction (using fixed or optimized step lengths; applying Conjugate Gradient or BFGS methods; implementing regularization).

## Hierarchy for verification

1. Do forward modeling for a single source and a single adjoint source for an homogeneous model; then check the SGT outputs.
2. Do forward modeling for several sources and adjoint simulations at all receiver location for those sources; heck the SGT outputs.
3. Perform iterative inversion from homogeneous half space to 1-D structure.

## To Do list

1. Modify the Emod3D code to have the required output wavefield. Convert existing Matlab code to python. Oct.24th-Nov.2nd.
2. Perform the forward modeling for synthetic models for the given numbers of sources and stations. Nov.02-Nov.09.
3. Calculate the adjoint source and perform the adjoint simulation. Calculate the sensitive kernels and iteratively update the velocity model. Nov.09-Nov.23.
4. Improve the computation (by making the code more independent of the forward modeling if possible) and documentation (including diagram for the new created functions). Nov.23-Dec.07.

## Verification

Can interrogation of the final inverted velocity model and a comparison to the true velocity model be the first method of verification?

Verifying the inverted result using FWT - Adjoint Wavefield method with FWT-Scattering Integral or SASW methods.

## \*References

1. Graves, R. W. ,1996. Simulating seismic wave propagation in 3D elastic media using staggered grid finite differences, *Bull. Seismol. Soc. Am.* 86, 1091–1106
2. Butzer, S., A. Kurzmann, and T. Bohlen, 2013, 3D elastic full-waveform inversion of small-scale heterogeneities in transmission geometry: *Geophysical Prospecting*, 61, 1238–1251.
3. Chen, P., Jordan, T.H. & Lee, E.-J., 2010. Perturbation kernels for generalized seismological data functionals (GSDF). *Geophysical Journal International*, 183(2), pp.869-83.
4. Tromp, J., Tape, C. & Liu, Q., 2005. Seismic tomography, adjoint methods, time reversal and banana-doughnut kernels. *Geophysical Journal International*, 160, pp.195-216.