The isotropic misfit kernels representing Fre'chet derivatives with respect to the bulk modulusκ, shear modulus μ and density ρ

Kernel for shear modulus, see e.g. Tromp et al. (2005), equation (17)

$$K_\mu(x) = -\int_0^T 2\mu(x) D^\dagger(x, T-t) : D(x,t) dt$$

Kernel for bulk modulus, see e.g. Tromp et al. (2005), equation (18)

$$K_\kappa(x) = -\int_0^T \kappa(x)[\bigtriangledown.s^\dagger(x, T-t)][\bigtriangledown.s(x,t)]dt$$

Kernel for density, see e.g. Tromp et al. (2005), equation (14)

$$K_\rho(x) = -\int_0^T \rho(x) s^\dagger(x, T-t)\ddot{s}(x,t)]dt$$

where D and $D^\dagger$ denote the traceless strain deviator and its waveform adjoint; s and $s^\dagger$ are the displacement fields and its adjoint waveform.
Alternatively, we may express the Fre'chet derivatives in terms of variations in shear wave speed V_s and compressional wave speed V_p

$K_{V_s}(x) = 2(K_\mu - \frac{4}{3}\frac{\mu}{\kappa}K_\kappa), K_{V_p}(x) = 2(\frac{\kappa+\frac{4}{3}\mu}{\kappa})K_\kappa.$

The isotropic misfit kernels representing Fre'chet derivatives with respect to the Lame parameters λ, μ and density ρ

Mora(1987) formulated the gradients for a second-order elastic wave equation in the time domain from the perturbation theory, see e.g. Butzer, et al. (2013), equations

(4-6):

$$K_\rho = \sum_s \int dt[\frac{\partial u_i}{\partial_t}\frac{\partial \Psi_i}{\partial_t}]$$

$$K_\lambda = -\sum_s \int dt[\frac{\partial u_i}{\partial x_i}\frac{\partial \Psi_j}{\partial x_j}]$$

$$K_\mu = -\frac{1}{2}\sum_s \int dt[(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})x(\frac{\partial \Psi_i}{\partial x_j} + \frac{\partial \Psi_j}{\partial x_i})]$$

where u_i is the i-th component of the forward propagated displacement.
The i-th component of the back-propagated wavefield Ψ_i in the time domain is defined as:

$$\Psi_i(x, x_s, T - t) = \sum_r \int d\tau G^{ip}(x, t, x_r, \tau)\delta u_p(x_r, T - \tau)$$

where $p = 1, 2, 3$; $i = 1, 2, 3$
This backward wavefield is generated at the receivers using the adjoint sources acting as a source time function. G^{ip} is the Green's function for displacement with p referring to the component of the adjoint source δu_p

We also can express the Fre'chet derivatives in terms of variations in shear wave speed V_s and compressional wave speed V_p

$K_{V_p} = 2\rho v_p K_\lambda, K_{V_p} = -4v_s\rho K_\lambda + 2v_s\rho K_\mu.$