Robust and efficient estimation of structural collapse capacity using the central difference time integration scheme

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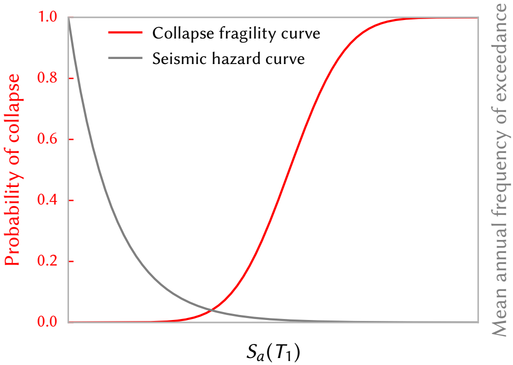
Background and Motivation

- Estimation of structural collapse capacity requires the numerical simulation of structural response under a number of intense ground motions that produce large inelastic deformations
- The numerical simulations commonly use implicit time integration schemes
  - They often fail to converge, especially when using long duration ground motions
  - Lots of execution time is spent in attempts to force convergence, which are not always successful
- The explicit central difference time integration scheme is proposed as a robust and efficient alternative

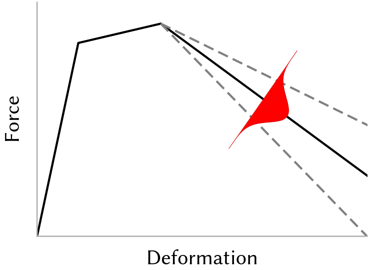
Structural collapse capacity



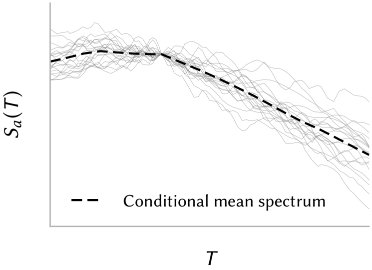
- Capacity of a structure to resist collapse under earthquake ground motion
- Defined by a *collapse fragility curve*, which quantifies the probability of collapse as a function of the ground motion intensity
- Gives the *mean annual frequency of collapse* when integrated with the seismic hazard curve
- Used in seismic design code calibration and loss assessment

Primary influencing factors

Solve the **right problem**



- Structural characteristics
  - Accuracy of structural model
  - Uncertainty in model parameters



- Ground motion characteristics
  - Hazard-consistent ground motions with appropriate response spectral shapes and durations
  - Uncertainty in characteristics of anticipated ground motions

Secondary influencing factors

Solve the **problem right**

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{f}(t) = -\boldsymbol{M}\boldsymbol{\iota}\ddot{\boldsymbol{u}}_{\boldsymbol{g}}(t)$$

- Numerical time integration scheme used
  - implicit schemes (e.g. Newmark average acceleration, HHT-α) often fail to converge, especially when using complex structural models and long duration ground motions
  - explicit schemes (e.g. central difference) are more robust, and preferred in analyses involving large nonlinear deformations, like blast and crash simulations; structural collapse simulations fall in the same category
- Criteria employed to detect structural collapse
- Analysis software (e.g. OpenSees, Perform 3D) and linear algebra solver (e.g. LAPACK, MUMPS, PETSc) used
  - treatment of ill-conditioned matrices at large nonlinear deformations
- Architecture of machine used to run the analysis
  - precision of computations

Newmark average acceleration vs. Central difference

Newmark average acceleration	Central difference

Newmark average acceleration

Implicit scheme

$$\left(\frac{4}{\Delta t^2}\boldsymbol{M} + \frac{2}{\Delta t}\boldsymbol{C}\right)\boldsymbol{u}_{i+1} = \boldsymbol{p}[\boldsymbol{M}, \boldsymbol{C}, \Delta t, \boldsymbol{u}_i, \dot{\boldsymbol{u}}_i, \ddot{\boldsymbol{u}}_i, (\ddot{\boldsymbol{u}}_g)_i] - \boldsymbol{f}_{i+1}$$

- Solves for equilibrium at end of time step
- Requires solution by iteration; convergence is not guaranteed
- If convergence fails
  - try other solution algorithms, e.g. Modified Newton-Raphson, Newton-Raphson with initial stiffness
  - try other implicit schemes with algorithmic damping
  - try reducing Δt
- These attempts are time-consuming
- If they all fail, structural collapse is declared even if collapse deformation threshold is not exceeded

Central difference

Explicit scheme

$$\left(\frac{1}{\Delta t^2}\boldsymbol{M} + \frac{1}{2\Delta t}\boldsymbol{C}\right)\boldsymbol{u}_{i+1} = \boldsymbol{p}[\boldsymbol{M}, \boldsymbol{C}, \Delta t, \boldsymbol{u}_i, \boldsymbol{u}_{i-1}, (\ddot{\boldsymbol{u}}_g)_i] - \boldsymbol{f}_i$$

- Solves for equilibrium at beginning of time step
- No iteration required
- If $\boldsymbol{C}$ is constant (and diagonal), matrix needs to be factorized only once
- Very amenable to parallelization by domain decomposition

Newmark average acceleration vs. Central difference

Newmark average acceleration	Central difference

Newmark average acceleration

Unconditionally stable

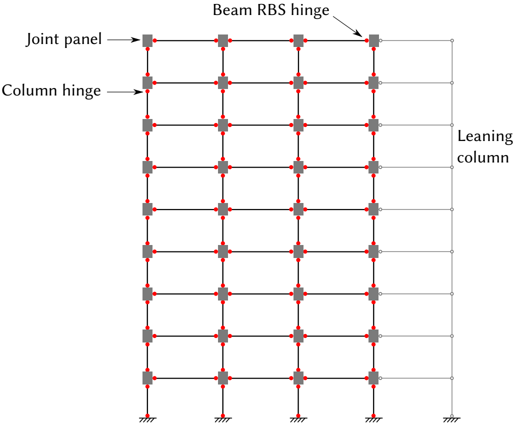
- Δt limited by accuracy, not stability
- Can use relatively large Δt ($\sim 10^{-3}$ s to 10^{-2} s), which is usually reduced upon encountering non-convergence
- Not easy to predict duration of analysis

Central difference

Conditionally stable

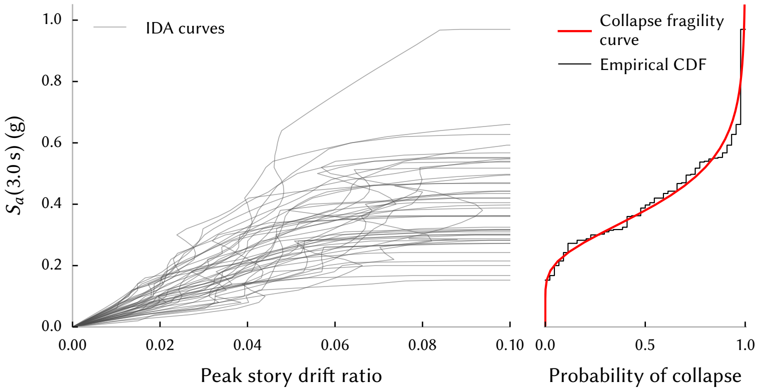
- $\Delta t \leq \frac{T_{min}}{\pi}$ for stability
- Δt used is usually relatively small ($\sim 10^{-4}$ s)
- T_{min} is usually unchanged in inelastic range
- Mass/moment of inertia must be assigned to all degrees of freedom
- Impractical to use rigid elements or penalty constraints
- Easy to predict duration of analysis (useful for parallel task scheduling)

Structural model



- 9-story steel moment frame building from SAC steel project
- 2d concentrated plastic hinge model created in OpenSees
- Plastic hinges follow Ibarra-Medina-Krawinkler bilinear hysteretic model
- Fundamental elastic modal period is 3.0 s
- Collapse capacity estimated separately using Newmark average acceleration and central difference schemes

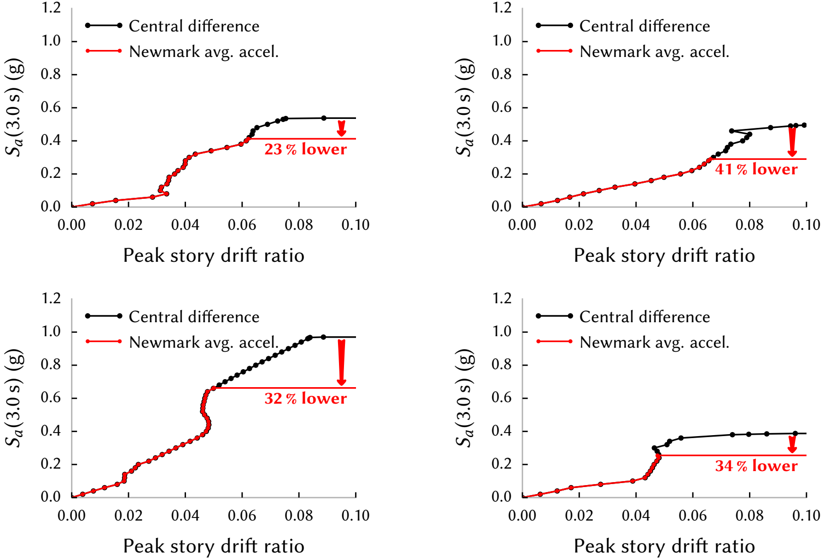
Incremental dynamic analysis (IDA)



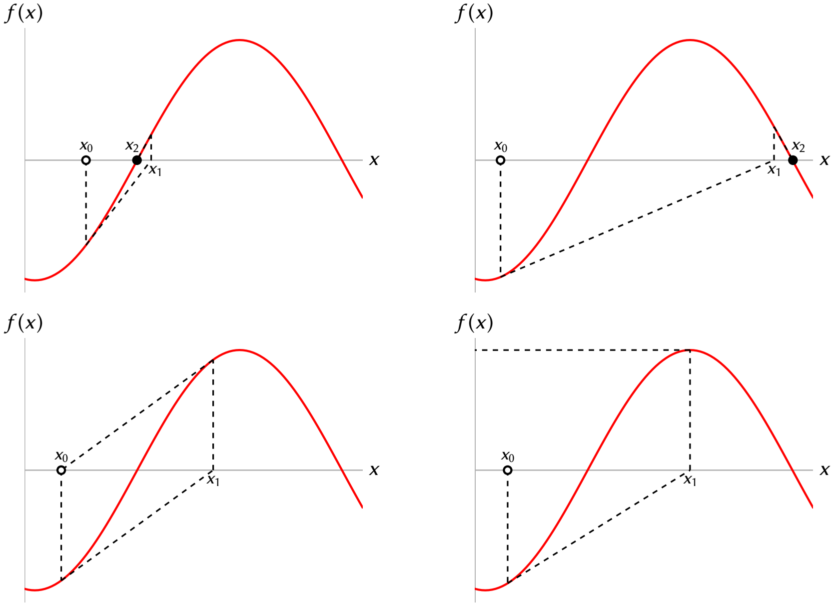
- Used FEMA P695 far field record set, containing 44 ground motions
- Each ground motion is progressively scaled to higher intensity levels until it causes structural collapse
- The collapse fragility curve is fit to the distribution of ground motion intensity levels at which structural collapse occurs

IDA curves bifurcate due to non-convergence

Difference in estimated collapse capacity > 10 % for **12** out of 44 ground motions

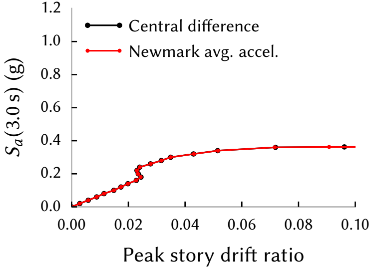


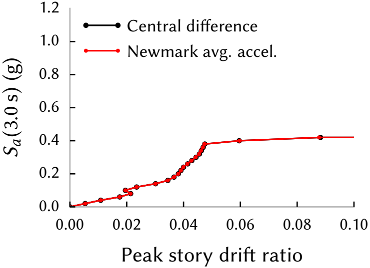
Possible outcomes of *full* Newton-Raphson algorithm

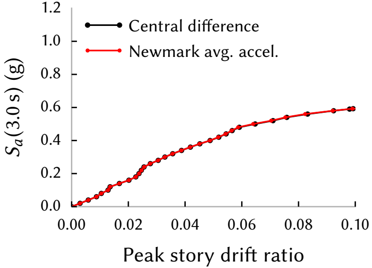


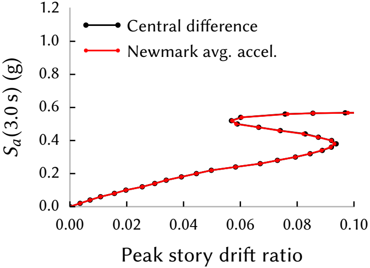
IDA curves are similar when analyses converge

Difference in estimated collapse capacity < 1 % for **29** out of 44 ground motions

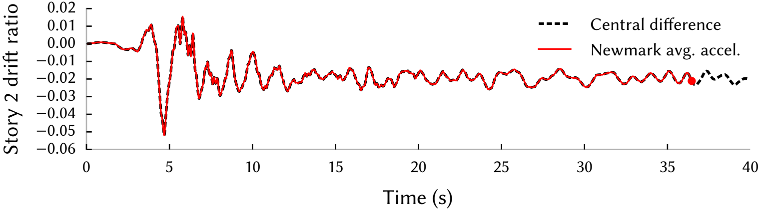




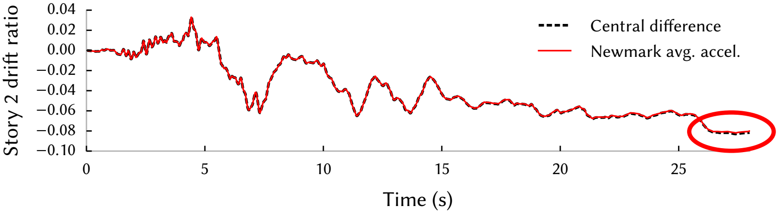




Comparison of representative time histories

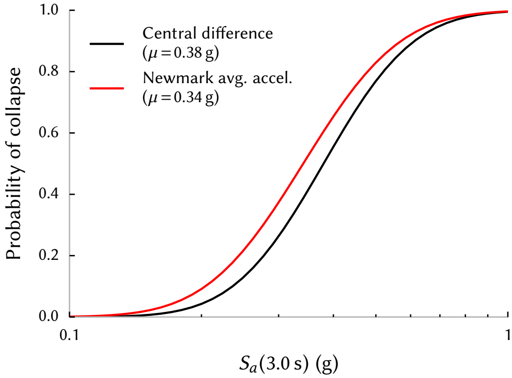


- Time histories are practically identical until the point of non-convergence, if any
- Could use implicit scheme until point of non-convergence and explicit scheme thereafter, but currently facing implementation issues in OpenSees



- Small differences are sometimes observed at large deformations (peak story drift ratio > 0.06)

Effect on estimated collapse fragility curves



- Median collapse capacity is under-estimated by 10 % when using the Newmark average acceleration time integration scheme
- Similar effect expected on collapse fragility curves estimated using multiple stripe analysis as well

Comparison of analysis runtimes

One analysis using one ground motion

Time integration scheme	**Rayleigh damping matrix**	**Type of solver**	Δt (s)	**Analysis runtime** (min)
Newmark avg. accel. low scale factor w/o convergence attempts	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	50×10^{-4}	1.0
Newmark avg. accel. high scale factor w/ convergence attempts	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	$\leq 50 \times 10^{-4}$	20.9
Central difference	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	1.5×10^{-4}	15.9
Central difference	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{initial}$	Sparse (*factor once*)	1.5×10^{-4}	3.3
Central difference	$\alpha \boldsymbol{M}$	Diagonal (*factor once*)	1.5×10^{-4}	2.9

- Using $\boldsymbol{K}_{initial}$ instead of $\boldsymbol{K}_{current}$ in the Rayleigh damping matrix has been shown to produce spurious damping forces
- Other option is to use a modal damping matrix, which is also constant

Comparison of analysis runtimes

Entire IDA (using 160 processors and dynamic load balancing)

Time integration scheme	**Rayleigh damping matrix**	**Type of solver**	Δt (s)	**IDA runtime** (min)
Newmark avg. accel.	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	$\leq 50 \times 10^{-4}$	118
Central difference	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{current}$	Sparse	1.5×10^{-4}	154
Central difference	$\alpha \boldsymbol{M} + \beta \boldsymbol{K}_{initial}$	Sparse (*factor once*)	1.5×10^{-4}	32
Central difference	$\alpha \boldsymbol{M}$	Diagonal (*factor once*)	1.5×10^{-4}	27

- Only 1 out of 632 total analyses conducted using the Newmark average acceleration scheme completed without any convergence errors
- 567 out of the 632 analyses completed using solution algorithms other than *full* Newton-Raphson
- 23 out of the 632 analyses completed after reducing Δt

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Conclusion

- Computational choices influence the estimated structural collapse capacity, in addition to structural and ground motion characteristics
- The explicit central difference time integration scheme is a robust and efficient alternative to commonly used implicit time integration schemes like Newmark average acceleration
- Advantages of the central difference scheme
  - Robust: not affected by convergence errors
  - Efficient: shorter runtimes despite using a smaller Δt
    - Most efficient when using constant (and diagonal) $\boldsymbol{C}$ matrix
    - Very amenable to parallelization
- Disadvantages of the central difference scheme
  - Mass/moment of inertia must be assigned to all degrees of freedom
  - Impractical to use rigid members or penalty constraints

Thank you!

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